Anisotropic composite model for layered rock mass based on characteristics of soft interfaces

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Based on the Drucker–Prager criterion, an anisotropic composite model of layered rock mass is proposed with the physico-mechanical characteristics of soft interface and rock matrix, and it can be used to describe the anisotropic characteristics of rock strength and deformation, as well as progressive failure or sliding characteristics. Then, the proposed model has been successfully established and embedded on the basis of the developing platform of ABAQUS and MATLAB, realising nonlinear numerical calculation. Through a typical example, the proposed model is verified. Finally, the proposed model is used to simulate a tunnel construction project, and the results show that this model can better explain the deformation and failure phenomenon of layered rock mass, and they are in accordance with engineering practice.

Keywords: Layered rock mass, Anisotropic constitutive model, Soft interfaces, Numerical simulation, ABAQUS

Introduction

Many geomaterials, such as sedimentary rocks, exhibit significant initial or inherent anisotropy, which is a type of common rock mass in practical engineering such as the field of transportation, tunnel construction, oil or gas storage and high-level radioactive waste repository. The concept of anisotropy effects caused by soft interface was first proposed by Jeager.[1] Since then, many authors did many experimental and theoretical studies for layered rock mass. These studies clearly reveal the directional dependence of strength and indicate that the maximum axial compressive strength is associated with configurations in which the soft interfaces are either parallel or perpendicular to the loading direction. At the same time, the minimum strength is typically associated with the failure along the soft interfaces, which corresponds to sample orientation within the range of 30–60°. The behaviour of layered rock mass is determined not only by the properties of the rock matrix, but mostly by the presence and properties of discontinuities or soft interfaces within the rock mass.[2,3] Continuity, orientation and frictional characteristics of the soft interfaces influence the deformability and strength of the layered rock masses. For the safety analysis of structures constructed in such geological formations, it is necessary to develop constitutive models which are able to account for the influence of anisotropy.

Various constitutive models have been proposed for the description of plastic deformation and failure in layered rock mass, which can be divided into three types. The first type of models is based on the concept of discontinuous soft interfaces.[4,5] These models provide a direct interpretation of material anisotropic properties, in which soft interfaces are set as joint element or discontinuity element in rock mass. However, it is generally difficult to use such models for complex engineering applications. The second family of models are continuous failure criterions and essentially empirical in nature, and they are obtained from direct extension of isotropic formulations by introducing the variation of some parameters with dip angle of soft interfaces or the orientation of maximum principal stress.[6–8] However, the formulation of such models cannot reflect the different yield and failure modes, and have no physical nature differences between rock matrix and soft interfaces. Other models are a non-continuous failure criterion based on the concept of macroscopic composite materials.[9,10] These models consider two types of failure in layered rock mass: rock matrix breaking and shear slip of soft interfaces. The main advantage of this approach is that the proposed models retain the mathematical rigour, and at the same time, has a clear physical meaning for the parameters, which is obtained easily from experiments. However, the formulation of such models is complex, and the numerical implement is not easy.

A great number of investigations have been performed to characterise deformation properties and failure conditions in layered rock mass, but the study of anisotropy strength criterion and constitutive model is not perfect for the complex-layered rock mass. By application the assumption of layered rock mass as macroscopic composite materials, an anisotropic constitutive model is proposed based on the failure criterion of Drucker– Prager, which is determined by their own properties of rock matrix and soft

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interfaces. Based on the secondary development platform of constitutive model in ABAQUS, this model is successfully developed and embedded. Through the typical examples, the proposed model is verified.

Constitutive relation of layered rock mass

The layered rock mass is considered as a special type of macroscopical composite material, which is made of isotropic rock matrix and a set of soft interfaces. The coordinate system of layered rock mass is shown in Fig. 1, in which the x–y plane is the bedding surface and the z-axis is normal direction of soft interfaces for local coordinate system. The incremental relation of stress–strain in the global coordinate system can be defined as[11]

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{el}} = \mathbf{C}\mathrm{d}\boldsymbol{\sigma} = [\mathbf{C}_{\mathrm{r}} + \mathbf{C}_{\mathrm{f}}]\mathrm{d}\boldsymbol{\sigma} \tag{1}$$

where $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{el}}$ is the elastic strain increment of layered rock, $\mathrm{d}\boldsymbol{\sigma}$ is the stress increment of layered rock and $\mathbf{C}$ is the total flexibility matrix composed of $\mathbf{C}_{\mathrm{r}}$ for rock matrix and $\mathbf{C}_{\mathrm{f}}$ for soft interfaces.

According to the stress continuity of rock matrix and soft interfaces in layered rock mass, equation (1) can be rewritten as

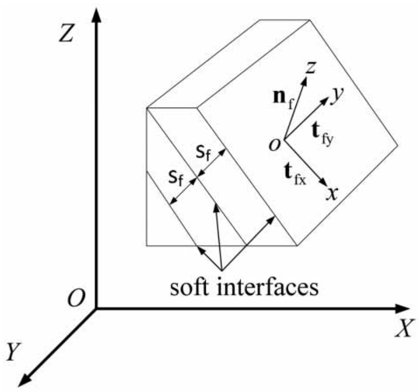
$$\begin{cases} \mathrm{d}\boldsymbol{\sigma}_{\mathrm{r}} = \mathrm{d}\boldsymbol{\sigma}_{\mathrm{f}} = \mathrm{d}\boldsymbol{\sigma} \\ \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{el}} = \mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{r}}^{\mathrm{el}} + \mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{f}}^{\mathrm{el}} \end{cases} \tag{2}$$

where $\mathrm{d}\boldsymbol{\sigma}_{\mathrm{r}}$ and $\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{r}}^{\mathrm{el}}$ are the stress and strain increment of rock matrix, $\mathrm{d}\boldsymbol{\sigma}_{\mathrm{f}}$ and $\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{f}}^{\mathrm{el}}$ are the stress and strain increment of soft interfaces.

According to the elasticity mechanics, the flexibility matrix of $\mathbf{C}_{\mathrm{r}}$and $\mathbf{C}_{\mathrm{f}}$ for three-dimensional problem in equation (1) can be defined as follows

$$\mathbf{C}_{\mathrm{r}} = \frac{1}{E} \begin{bmatrix} 1 & -\mu & -\mu & 0 & 0 & 0 \\ -\mu & 1 & -\mu & 0 & 0 & 0 \\ -\mu & -\mu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\mu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\mu) \end{bmatrix} \tag{3}$$

$$\mathbf{C}_{\mathrm{f}} = L\bar{C}_{\mathrm{f}}L^{T} \tag{4}$$



1 Coordinate system of layered rock mass

where E and μ are the elastic modulus and Poisson's ratio of rock matrix, L is the coordinate transformation matrix and $\bar{C}_{\mathrm{f}}$ is the flexibility matrix of soft interfaces in the local coordinate system defined as

$$\bar{C}_{\mathrm{f}} = \frac{A}{S} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/K_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/K_{sx} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/K_{sy} \end{bmatrix} \tag{5}$$

$$L = \begin{bmatrix} l_1^2 & l_2^2 & l_3^2 & 2l_1l_2 & 2l_2l_3 & 2l_3l_1 \\ m_1^2 & m_2^2 & m_3^2 & 2m_1m_2 & 2m_2m_3 & 2m_3m_1 \\ n_1^2 & n_2^2 & n_3^2 & 2n_1n_2 & 2n_2n_3 & 2n_3n_1 \\ l_1m_1 & l_2m_2 & l_3m_3 & l_1m_2+l_2m_1 & l_2m_3+l_3m_2 & l_3m_1+l_1m_3 \\ m_1n_1 & m_2n_2 & m_3n_3 & m_1n_2+m_2n_1 & m_2n_3+m_3n_2 & m_3n_1+m_1n_3 \\ n_1l_1 & n_2l_2 & n_3l_3 & n_1l_2+n_2l_1 & n_2l_3+n_3l_2 & n_3l_1+n_1l_3 \end{bmatrix} \tag{6}$$

where A and S are the fracture connectivity rate and fracture spacing, K_n is the normal stiffness, K_{sx} and K_{sy} are the tangential stiffness in the bedding surface, (l_1, m_1, n_1) is the direction cosine of x-axes to the global coordinate system, (l_2, m_2, n_2) is the direction cosine of y-axes to the global coordinate system, and (l_3, m_3, n_3) is the direction cosine of z-axes to the global coordinate system.

The plastic strain increment can be calculated from to the plastic potential function as

$$d\boldsymbol{\varepsilon}^{\mathrm{pl}} = \lambda\frac{\partial G}{\partial \boldsymbol{\sigma}} \tag{7}$$

where λ is plastic multiplier, G is plastic potential function.

Based on the above analysis, the total strain of layered rock mass can be written as[12]

$$\mathrm{d}\boldsymbol{\varepsilon} = \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{el}} + \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{pl}} = [\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{r}}^{\mathrm{el}} + \mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{f}}^{\mathrm{el}}] + [\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{r}}^{\mathrm{pl}} + \mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{f}}^{\mathrm{pl}}] \tag{8}$$

where $\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{r}}^{\mathrm{pl}}$ and $\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{f}}^{\mathrm{pl}}$ are plastic strain increment of rock matrix and soft interfaces, respectively.

Anisotropic strength criteria of layered rock mass

The strength characteristics of layered rock mass are the comprehensive reflection of rock matrix and soft interfaces owing to the different mechanical properties, and it is possible that failure or damage occurs in the rock matrix or soft interfaces.

The normal stress and shear stress on the soft interfaces can calculated as

$$\begin{cases} p_{\mathrm{f}} = \mathbf{n}_{\mathrm{f}} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}_{\mathrm{f}} \\ \tau_{\mathrm{fi}} = \mathbf{n}_{\mathrm{f}} \cdot \boldsymbol{\sigma} \cdot \mathbf{t}_{\mathrm{fi}} \end{cases} \tag{9}$$

where p_{f} and $\tau_{\mathrm{f}i}$ are the normal stress and shear stress on the soft interfaces, $\mathbf{n}_{\mathrm{f}}$ is the normal unit vector of soft interfaces, $\mathbf{t}_{fi}(\mathbf{t}_{fx}, \mathbf{t}_{fy})$ are two mutually perpendicular unit vectors on the soft interfaces (Fig. 1), and $\boldsymbol{\sigma}$ is stress tensor in the layered rock mass.

The normal stiffness K_n tends to zero when the normal stress on the soft interfaces becomes the state of tensile stress ($p_f \le 0$).

Similarly, the normal strain and shear strain of soft interfaces can be expressed as

$$\begin{cases} \varepsilon_{fn} = \mathbf{n}_f \cdot \boldsymbol{\varepsilon} \cdot \mathbf{n}_f \\ \gamma_{fi} = \mathbf{n}_f \cdot \boldsymbol{\varepsilon} \cdot \mathbf{t}_{fi} + \mathbf{t}_{fi} \cdot \boldsymbol{\varepsilon} \cdot \mathbf{n}_f \end{cases} \tag{10}$$

where ε_{fn} and γ_{fi} are the normal strain and shear strain of soft interfaces, and $\boldsymbol{\varepsilon}$ is the strain tensor in the layered rock mass.

Based on the modified Drucker–Prager criterion, the yield function of soft interfaces can be defined as[10]

$$F_f = \tau_f - p_f \tan \phi_f - c_f \tag{11}$$

where ϕ_f and c_f are the friction angle and cohesion of soft interfaces, and τ_f is the total shear stress on the soft interfaces defined as

$$\tau_f = \sqrt{\tau_{fi}\tau_{fi}} \tag{12}$$

Similarly, the plastic potential function of soft interfaces can be written as

$$G_f = \tau_f - p_f \tan \varphi_f \tag{13}$$

where φ_f is the dilatancy angle of soft interfaces.

The shear failure occurs on the soft interfaces when the stress condition reaches critical state of $F_f = 0$, and the increment of equivalent plastic strain can be defined as

$$d\bar{\varepsilon}_f^{pl} = \sqrt{\frac{2}{3}}\left(d\boldsymbol{\varepsilon}_f^{pl}:\boldsymbol{\varepsilon}_f^{pl}\right) \tag{14}$$

where $d\boldsymbol{\varepsilon}_f^{pl}$ is the plastic strain of soft interfaces calculated by equation (7).

The plastic strain component on soft interfaces can be expressed as

$$\begin{cases} d\varepsilon_{fn}^{pl} = d\bar{\varepsilon}_f^{pl} \sin \varphi_f \\ d\gamma_{fi}^{pl} = d\bar{\varepsilon}_f^{pl} \dfrac{\tau_{fi}}{\tau_f} \cos \varphi_f \end{cases} \tag{15}$$

where $d\varepsilon_{fn}^{pl}$ is the normal plastic strain on soft interfaces, $d\gamma_{fi}^{pl}$ is the shear plastic strain on soft interfaces ($i = x, y$).

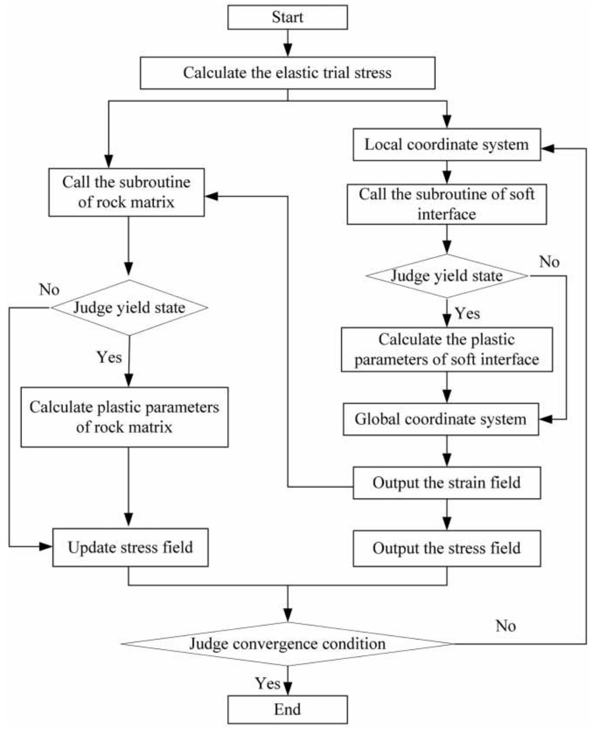
The yield function and potential function of rock matrix by Drucker–Prager criteria are defined as follows

$$\begin{cases} F_r = q - p \tan \phi_r - c_r \\ G_r = q - p \tan \varphi_r \end{cases} \tag{16}$$

where q is Mises stress, p is confining stress, c_r and ϕ_r are the cohesion and friction angle of rock matrix, and φ_r is the dilatancy angle of rock matrix.

Model implementation and numerical verification

As already described in sections 2 and 3, the layered rock is seen as a composite materials composed of rock matrix and soft interfaces. Here, we present the development and implementation of the above anisotropic constitutive model into the geomechanical simulator ABAQUS–MATLAB, in which MATLAB language is as the program platform and ABAQUS software is calculation solver for rock matrix and soft interfaces.



2 Flow chart of the secondary development of anisotropic constitutive model

There are four major parts in this program: rock matrix analysis part, soft interface part, MATLAB interface part and alternating iteration part.[13] The rock matrix part and soft interface part are executed on the compatible numerical grids and linked through the stress field by the alternating iteration method (Fig. 2).

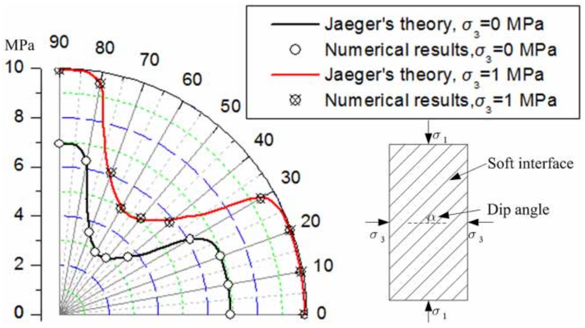
The calculation process can be depicted as follows: first, the rock matrix part is applied, then the soft interface part is called by taking the stress result of rock matrix as the initial stress condition, then the above rock matrix part is called by taking the stress result of soft interfaces as the initial stress condition, and the program is terminated until the stress difference is satisfied the condition of $||\sigma_r| - |\sigma_f|| \le \delta$ at any node in the numerical grids.

In order to verify the effectiveness of the above constitutive model, a typical triaxial compression test is applied to verify it. The mechanical parameters of rock matrix and soft interface in composite rock mass are shown in Table 1. The angle between the compressive load and soft interfaces is shown as Fig. 3.

Figure 3 shows the results of compressive strength for dip angle changes under the condition of triaxial compression. In this simulation, the compressive strength is affected obviously by dip angle of soft interfaces within the range [30° 80°] and tends to the minimum when the

Table 1 Mechanical parameters of calculated model

Material	***E*** (MPa)	μ	***c*** (MPa)	ϕ (°)
Rock matrix	800	0·3	2	30
Soft interface	…	…	1	20



3 The relation of strength and dip angle of layered rock

dip angle is 50–60°, whereas the compressive strength is not changed with the dip angle out of the range [30° 80°]. The numerical solution is consistent with the analytical solution from Jaeger's theory for strength characteristics, which demonstrates that the proposed model can simulate the anisotropic characteristics of layered rock mass.[11]

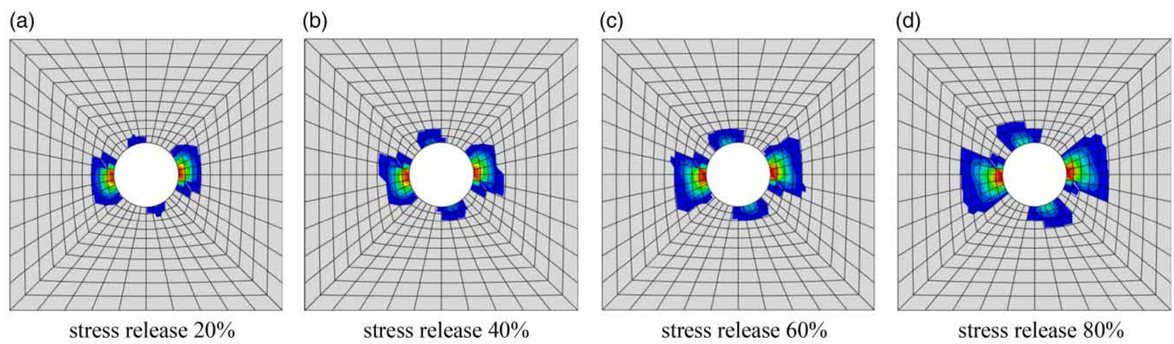
Cases analysis

A tunnel excavation process in layered rock mass is taken as an example, and the excavation disturbed zone (EDZ) is studied by the above anisotropic constitutive model. The over excavation radius of the investigated tunnel is 2·445 m, and the outer radius of lining is 2·4 m with thickness of 0·4 m. The *in situ* stress and the lateral pressure coefficient is 0·8. The dip angle of soft interfaces is 60°, and the spacing of soft interfaces is 0·3 m with the connectivity rate of 1 in the finite element model. The elastic parameters of soft interfaces are normal stiffness of 100 MPa m^{-1} and tangential stiffness of 60 MPa m^{-1}. The main calculation parameters are shown in Table 2.

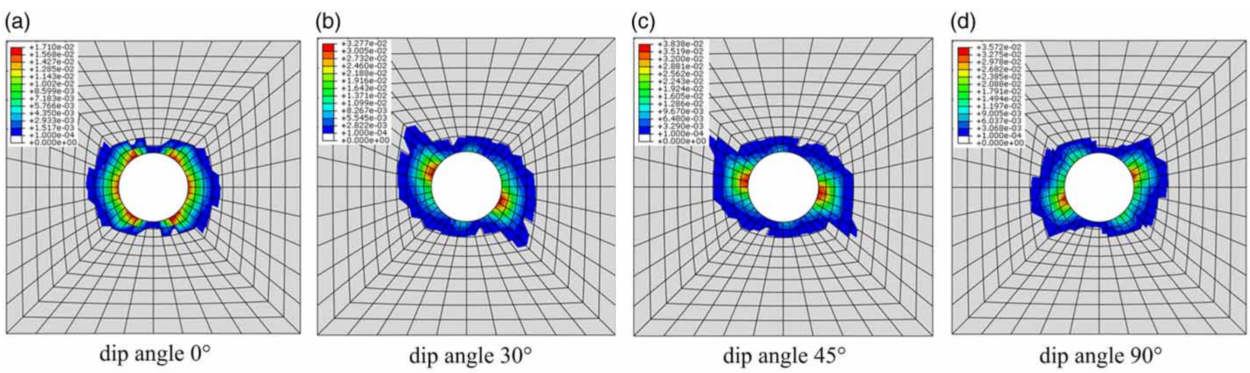
Owing to highly non-linear characteristics of layered rock mass, the excavation project and constitutive quality of tunnel have obvious effects on the EDZ. The influence of stress release on interface failure is studied by anisotropic constitutive model with stress release rates of 20, 40, 60 and 80%, respectively. The interface plastic strain variation plots are shown in Fig. 4. The magnitude and scope of plastic strain increase obviously with the stress releasing rate increasing. The maximum plastic strain is $2{\cdot}025\times10^{-2}$ with the stress release rate of 20%, whereas the maximum plastic strain is $5{\cdot}796\times10^{-2}$ with the stress release rate of 80%. The stability of the surrounding rock is mainly determined by the action of soft interfaces, where the magnitude of plastic strain along soft interfaces is significantly greater than that of rock matrix.

Table 2 Rock mass mechanical parameters

Name	Weight density (kN m^{-3})	Elastic modulus (MPa)	Poisson's ratio	Cohesion (MPa)	Friction angle (°)	Dilatancy angle (°)
Rock matrix	23·0	300	0·13	0·4	28·16	0
Soft interface	…	…	…	0·04	15	0
Lining	25	30 000	0·25	…	…	…



4 Distribution graph of plastic zone of soft interfaces in different release ratio



5 Distribution graph of plastic zone with different dip angle of soft interfaces

The dip angle of soft interfaces has significant influence on the stability of underground surrounding rock. This is illustrated in Fig. 5 when the stress release rate is 60%. It can be seen that the distribution state of EDZ is controlled by the dip angle of soft interfaces, and the direction of long axis of EDZ is against that of dip angle. When the direction of soft interfaces is not horizontal or vertical, the distribution graph of total plastic zone is asymmetric. The dip angle of soft interfaces obviously affects the maximum of plastic strain in surrounding rock. The damage level of surrounding rock is lowest for the dip angle of 0°, whereas the damage level of surrounding rock is highest for the dip angle of 60°.

Conclusions

The main conclusions that can be drawn from this study are

1. Based on Drucker–Prager criterion, a new anisotropic constitutive model is provided for layered rock mass, which can consider the own mechanics properties of rock matrix and soft interfaces.
2. The proposed constitutive model is implemented in the simulator of ABAQUS–MATLAB, which the ABAQUS software is embedded as solver. A typical example of compression is tested, and the numerical results are consistent with the analytic solution of Jaeger theory.
3. The proposed model is applied to tunnel excavation simulation, and the results show that it can effectively reflect the intrinsic anisotropy of layered rock and can depict the deformation and failure of rock matrix and soft interfaces, respectively.

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