GEOMECHANICS

# A New Approach to Determining Multistage Hydraulic Fracture Size by Well Production Data

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**Abstract**—The simplified two-zone model to describe fluid flow in steady-state horizontal holes with multistage hydraulic fracturing is proposed, and the analytical solution in order to calculate sizes of hydrofractures is presented. The new procedure to determine size of multistage hydrofractures in horizontal holes is tested using field data on China deposits. The calculation results prove the efficiency of the procedure in the determination of steady-state conditions. The procedure can be used in optimization of hydraulic fracturing.

Keywords: Horizontal hole, multistage hydraulic fracturing, fracture size, field data analysis.

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## INTRODUCTION

Most of new oil and gas fields currently being put intoproduction are characterized by complicated porosity and permeability properties (P&P). A combination of the technologies of horizontal well (HD/HW) drilling with multistage hydraulic fracturing (MSHF) is invited for more effective development of such fields. During interpretation of the HD MSHF field data using the standard method presented in [1], the encountered problems are basically described as: field data are insufficient due to technical reasons and/ or inappropriate for practical applications (e.g., the data are either too scattered, or discrete). As such, the problems can be solved using the new simplified model describing fluid flow under steady-state conditions in reservoirs with two zones having different P&P properties.

The improved analytical solution resulting from [2], was used for calculating the size of MSHF fractures. The application of the proposed technique to determining sizes of MSHF fractures in HW is illustrated by an example of using field data from a field operated in PRC. The proposed model can also be applicable for analyzing the effect of maximum amount of fractures on production rate of a well and for optimization of hydraulic fracturing.

# 1. PROBLEM FORMULATION

The assumptions accepted in the course of fluid flow schematization in HW with MSHF (Fig. 1a) are: HW is placed in the reservoir with a strip-shaped drainage region, persistent in thickness and saturated with a single-phase liquid; the roof and base of the layer are impermeable; vertically, reservoir permeability is insignificant; the fluid flow regime in zone I is one-dimensional plane-linear fluid flow oriented to transverse MSHF fractures perpendicular to the wellbore, and plane-linear flow to the boundary between zones I and II in Zone II; there is no pressure drop neither along the horizontal wellbore, nor in fractures; the HW flow rate is defined as sum of the flows in all fractures (total flux rate).



Fig. 1. The fluid flow scheme for HW with transverse hydraulic fractures.

## 2. MATHEMATICAL MODEL OF THE PROBLEM

**Zone I.** The fluid influx filling a half of the fracture in a segment of zone I (Fig. 1b), is calculated by formula

$$q_1 = -\frac{k_1}{\mu B} \iint \frac{\partial p}{\partial x} \, dS \,, \tag{1}$$

where  $k_1$  is the permeability of zone I;  $\mu$  is the oil viscosity; B is the formation volume factor; S is the cross-sectional area normal to fluid; dS = dzdy.

In this paper, we considered the steady-state flow for the simplicity principle. Suppose that at the boundary between Zones I and II at a point equidistant from the adjacent fractures, the pressure will be equal to a value  $p_0$ , which is lesser than the initial reservoir pressure (Fig. 1c).

Results from approximate calculations showed the pressure function distribution in Zone I are presented in the following form [2]:

$$p(x=0, y) = p_{wf} + \frac{p_0 - p_{wf}}{x_f} y, \qquad (2)$$

$$p(x, y) = p(x = 0, y) - \frac{p(x = 0, y) - p_{wf}}{x_s/2} x, \qquad (3)$$

where  $p_{wf}$  is the bottomhole pressure;  $p_0$  is the pressure at the boundary between Zones I and II at a point equidistant from the adjacent fractures;  $x_f$  is the half-length of fractures;  $x_s = L/(N-1)$ is the distance between fractures, L is the horizontal wellbore section, N is the quantity of fractures.

Consequently, the pressure distribution function has the form:

$$p(x, y) = p_{wf} + \left(1 - \frac{2x}{x_s}\right) \frac{p_0 - p_{wf}}{x_f} y .$$
(4)

Note that in relation (4), given the cyclic character of computations in case where the number of fractures is greater than one, the variable can take values from 0 to  $x_s/2$ . Figure 2 shows the pressure distribution in three-dimensional space in accordance with equation (4).

The fluid influx half-filling fracture in the segment is obtained after integrating:

$$q_{1} = \frac{k_{1}h}{\mu x_{s}B} (p_{0} - p_{wf}) x_{f}, \qquad (5)$$

where *h* is the reservoir thickness.



Fig. 2. Three-dimensional pressure distribution (Zone I).

The total flow rate of HW with MSHF can thus be represented as follows:

$$Q_{1} = 4q_{1}(N-1) + Q_{D} = (N-1)^{2} \frac{4k_{1}h}{\mu LB} (p_{0} - p_{wf})x_{f} + Q_{D};$$
(6)

here,  $Q_1$  is the influx in HW with MSHF;  $Q_D$  is the external influx approaching first and last fractures (Fig. 3).

In [3], the relation obtained on the basis of the well-known Dupuy formula allows calculating this flow rate:

$$Q_{D} = \frac{2\pi kh(p_{k} - p_{wf})}{\mu B \left[ \ln \left( \frac{R_{k}}{r_{w}} \right) + s \right]},$$
(7)

where k is the reservoir permeability;  $p_k$  is the boundary pressure;  $R_k$  is the external boundary radius;  $r_w$  is the borehole radius; s is the pseudoskin factor.

A pseudoskin factor was introduced in [3], allowing to take into account the diversity of specific inflows to wells with a more complex profile (horizontal, multilateral, with hydraulic fracturing, etc.) rather than in the vertical wells alone.

**Zone II.** It is assumed that the pressure distribution along the x-axis at the interface between Zones I and II of the flow region (Fig. 4c) is, by itself, a linear function:

$$p(x, y=0) = p_0 - \frac{p_0 - p_{wf}}{x_s/2} x.$$
(8)



Fig. 3. Influx  $Q_D$  diagram (with the only fracture throughout the entire wellbore length).

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Fig. 4. Fluid flow diagram for HW with transverse hydraulic fractures (Zone II).

The fluid influx from the drainage zone boundary to transverse fractures in Zone II (Fig. 4b) is written as

$$q_2 = -\frac{k_2}{\mu B} \iint \frac{\partial p}{\partial y} dS , \qquad (9)$$

where  $q_2$  is the fluid influx to fractures in the segment of Zone II;  $k_2$  is the permeability of Zone II; dS = dxdz;  $\partial p / \partial y = [p_k - p(x, y = 0)]/l$ ,  $l = R_k - x_f$  is a distance from the drainage area boundary to the boundary of Zone II.

Figure 5 shows the pressure distribution in Zone II, based on the foregoing assumptions.

In accordance with the said assumption, the influx to the horizontal well with hydraulic fractures can be written as:

$$Q_2 = 4(N-1)q_2 + Q_D = 2\frac{k_2hL}{\mu lB} \left( p_k - \frac{p_0}{2} - \frac{p_{wf}}{2} \right) + Q_D, \qquad (10)$$

where  $Q_2$  is the fluid influx from the drainage area boundary to transverse fractures.

Given that the fluid flow in Zones I and II is steady, the production rates calculated from relations (6) and (10) will be equal. Equating these relations, we obtain:

$$p_{0} = \frac{p_{k} - \left[\frac{1}{2} - (N-1)^{2} \frac{2x_{f}l}{L^{2}}\lambda\right] p_{wf}}{\frac{1}{2} + (N-1)^{2} \frac{2x_{f}l}{L^{2}}\lambda},$$
(11)

where  $\lambda = k_1 / k_2$  is the relationship between permeabilities of Zones I and II.



Fig. 5. Three-dimensional pressure distribution (Zone II).

When applying formula (11) to (6), we obtain:

$$x_{f} = \frac{1}{4(N-1)^{2} \left(\frac{2k_{1}h}{\mu BL} \frac{p_{k} - p_{wf}}{Q_{1} - Q_{D}} - \frac{\lambda l}{L^{2}}\right)}.$$
(12)

In (12), by substituting parameter *l* for  $R_k - x_f$  we have:

$$x_{f} = \frac{1}{4(N-1)^{2} \left(\frac{2k_{1}h}{\mu BL} \frac{p_{k} - p_{wf}}{Q_{1} - Q_{D}} - \frac{\lambda R_{k}}{L^{2}}\right) + \frac{4(N-1)^{2}\lambda}{L^{2}} x_{f}}.$$
 (13)

This expression can be used to calculate the half-length of fractures with known values of the parameters present in (13). If we write (13) in other form, we can determine the well flow rate and analyze its dependence on the half-length of the fractures and their amount, which enables optimization of the MSHF design solutions. With gas flowing in the reservoir, formula (13) should be rewritten in a different form, taking into account drops in the pseudo-pressure, and replacing parameter  $\mu B/2$  with  $p_{sc}T/T_{sc}$ . In this case, the pseudo-pressure is:

$$m(p) = 2 \int \frac{p}{\mu_g z} dp , \qquad (14)$$

 $\mu_{\rm g}$  is the gas viscosity; z is the gas supercompressibility factor.

The pseudo-pressure drop can be calculated with average values of gas supercompressibility factor and viscosity:

$$m(p_k) - m(p_{wf}) = 2 \int_{p_{wf}}^{p_k} \frac{p}{\mu_g z} dp = \frac{p_k^2 - p_{wf}^2}{\overline{\mu}_g \overline{z}}.$$
 (15)

Here,  $\overline{\mu}_g = \mu(p_k + p_{wf})/2$ —is average gas viscosity value;  $\mu(p)$ —is fluid viscosity to pressure ratio;  $\overline{z} = z(p_k + p_{wf})/2$ —is average value of gas supercompressibility factor; z(p)—gas supercompressibility factor to pressure ratio [4].

Thus, the formula for calculating the half-length of fractures under conditions of gas flow is written as follows:

$$x_{f} = \frac{1}{4(N-1)^{2} \left(\frac{T_{sc}k_{1}h}{Tp_{sc}L} \frac{p_{k}^{2} - p_{wf}^{2}}{(Q_{g} - Q_{D})\overline{\mu}_{g}\overline{z}} - \frac{\lambda R_{k}}{L^{2}}\right) + \frac{4(N-1)^{2}\lambda}{L^{2}}x_{f}},$$
(16)

where  $Q_g$  is the gas flow rate at surface conditions;  $p_{sc}$  is the pressure in steady state; T is the reservoir temperature;  $T_{sc}$  is the temperature in steady state.

To calculate the fracture half-length, considering nonlinearity of relation (16), the following modification with the new parameters A and B can be used:

$$x_{f} = \frac{A}{2B} \left( \sqrt{1 + \frac{4B}{A^{2}}} - 1 \right), \tag{17}$$

where 
$$A = 4(N-1)^2 \left( \frac{T_{sc}k_1h}{Tp_{sc}L} \frac{p_k^2 - p_{wf}^2}{(Q_g - Q_D)\overline{\mu}_g \overline{z}} - \frac{\lambda R_k}{L^2} \right); B = \frac{4(N-1)^2 \lambda}{L^2}$$

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Fig. 6. Dynamics of wellhead pressure (1) and flow rates (2) with multi-stage hydraulic fracturing of the formation.

#### 3. FIELD DATA INTERPRETATION BY THE EXAMPLE OF GAS FIELD IN PRC

When processing and interpreting the production data from horizontal wells with multi-stage hydraulic fracturing of the reservoir in a PRC field, it was found that the obtained data are too scattered (Fig. 6), which makes it impossible to identify different flows and determine the parameters of fractures using the standard methodology presented in [1]. From Fig. 6 it follows that in the period between the 95th and 170th days, the wellhead pressure is found to be almost steady, which suggests stationary influx during this period. In turn, with this assumption it is possible to estimate the half-length of fractures  $x_f$ , measuring 153.5 m according to the proposed method. The well bottomhole

pressure can be calculated using the formula from [5], with account of the wellhead pressure.

The following parameters of HW with MSHF and the in-situ fluid (at  $Q_D = 0.14 \cdot 10^4 \text{ m}^3/\text{day}$ ) have been established: initial reservoir pressure—28.5 MPa; reservoir temperature—104°C; reservoir thickness—20 m; the horizontal segment length—959 m; quantity of fractures—6; gas viscosity— 0.019 mPa s; gas supercompressibility factor—0.98; well radius—0.09 m; reservoir permeability— 0.015  $\cdot 10^{-3} \text{ m/cm}^2$ ; depth of well penetration—3200 m.

## CONCLUSIONS

A simplified model for the fluid flow in a horizontal well with multi-stage hydraulic fracturing of formation under steady-state conditions is proposed, which served as a basis for the developed method for preliminary estimation of sizes of the MSHF-induced fractures. Given the paucity of the required information for the field data interpretation, the new technique can be used as an alternative for obtaining more accurate approaches. The paper provides results of the production data analysis and interpretation by the example of a field operating in People's Republic of China, illustrating determination of the half-length of the MSHF-induced fractures in HW. The obtained analytical solution can be applied to the analysis of the influence of the amount of fractures on the well production rate and to optimization of the quantity and extent of hydraulic fractures when designing HW with MSHF.

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